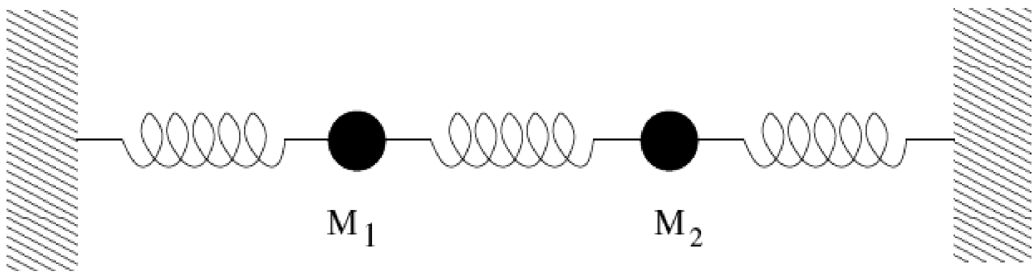
LSU Dept. of Physics and Astronomy
Qualifying Exam
Classical, Thermo, and Statistical Mechanics Question Bank
(01/2017)

1. A particle is dropped into a hole drilled straight through the center of the earth. Neglecting rotational effects, show that the particle's motion is simple harmonic. Compute the period and give an estimate in minutes. Compare your result with the period of a satellite orbiting near the surface of the earth.

2. Two identical bodies of mass m are attached by identical springs of spring constant k as shown in the figure.



   a) Find the frequencies of free oscillation of this system.

   b) Mass number 1 is displaced from its position by a small distance a_1 to the right while the mass number 2 is not moved from its position. If the two masses are released with zero velocity, what is the subsequent motion of mass number 2.

3. A planet is in circular motion about a much more massive star. The star under goes an explosion where three percent of its mass is ejected far away, equally in all directions. Find the eccentricity of the new orbit for the planet.

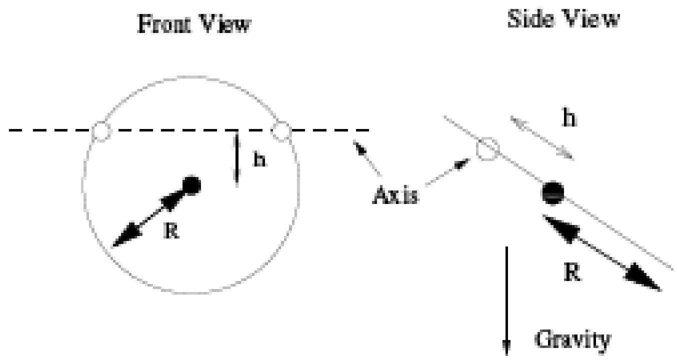
4. A homogeneous cube each edge of which has a length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Find the angular velocity of the cube when one face strikes the plane if:

   a) the edge cannot slip on the plane.

   b) sliding can occur without friction.

5. A chain of linear density μ (g/cm) is hanging vertically above a table. Its lowest point is at a height h above the table. The chain is released and allowed to fall. Calculate the force exerted on the table by the chain when a length x of chain has fallen onto the surface of the table.

6. A thin circular ring of radius R and mass M is constrained to rotate about a horizontal axis passing through two points on the circumference. The perpendicular distance from the axis to the center of the ring is h.



a) Find a Lagrangian for this object.

b) Find the period of small oscillations about this axis.

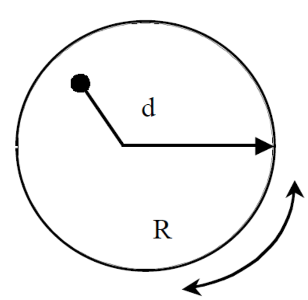
c) For what value of h is the period a minimum?

7. An object is dropped from a tower of height h. The tower is located at the equator of the earth. The rotational speed of the earth is Ω.

a) If the acceleration of gravity on the earth ignoring rotation is g, what is the observed acceleration due to gravity on the equator?

b) Even though it is released from rest, this object will not land directly below the point it is dropped. Calculate the amount and direction (N, E, S, W, or elsewhere) of the horizontal deflection of the object. You may assume the deflection is small.

8. Consider a pendulum formed by suspending a uniform disk of radius R at a point a distance d from its center. The disk is free to swing only in the plane of the picture.

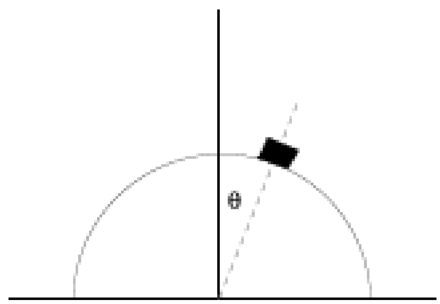


a) Using the parallel axis theorem, or calculating it directly, find the moment of inertia I for the pendulum about an axis a distance d $(0 \le d < R)$ from the center of the disk.

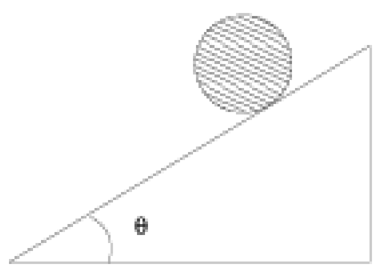
b) Find the gravitational torque on the pendulum when displaced by an angle ϕ.

c) Find the equation of motion for small oscillations and give the frequency ω. Further find the value of d corresponding to the maximum frequency, for fixed R and m.

9. A heavy particle is placed close to the top of a frictionless vertical hoop and allowed to slide down the loop. Find the angle at which the particle falls off.



10. A solid sphere of radius R and mass M rolls without slipping down a rough inclined plane of angle θ. Take the coefficient of static friction to be μ_s and calculate the linear acceleration of the sphere down the plane, assuming that it <u>rolls without slipping</u>. Calculate the maximum angle, $\theta_{\max}$, for which the sphere will not slip.



11. A rocket is filled with fuel and is initially at rest. It starts moving by burning fuel and expelling gases with the velocity u, constant relative to the rocket. Determine the speed of the rocket at the moment when its kinetic energy is largest.

12. A small lead ball of mass m is attached to one end of a vertical spring with the spring constant k. The other end of the spring oscillates up and down

with the amplitude A and frequency ω. Determine the motion of the ball after a long period of time. You may assume that the ball is subject to a small amount of damping, he damping force being given by $F = -bv$. Explain the answer.

13. In movie cameras and projectors film speed is 24 frames/second. You see on the screen a car that moves without skidding, and know that the real-life diameter of its wheels is 1m. The wheels on the screen make 3 turns per second. What are possible speeds of the car, assuming it is not moving in excess of 200 mile/hr? Investigate all possibilities.

14. A ball of mass m collides with another ball of mass M at rest. The collision is elastic and the motion is one dimensional. The ratio of the masses is a = M/m.

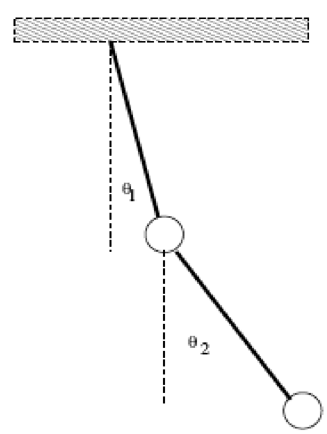
a) Determine how the energy lost by the moving ball depends on the mass ratio a, and find the value of a for which the energy loss is largest. Describe what happens at that value of the mass ratio.

b) Investigate the limiting cases of heavy and light balls and comment on your result.

15. Two particles of mass m_1 and m_2 move in circular orbit around each other under the influence of gravity. The period of the motion is T. They are suddenly stopped and then released, after which they fall towards each other. If the objects are treated as mass points, find the time they collide in terms of T.

16. Three points masses of identical mass are located at (a, 0, 0), (0, a, 2a) and (0, 2a, a). Find the moment of inertia tensor around the origin, the principal moments of inertia, and a set of principal axes.

17. Consider a double pendulum consisting of a mass m suspended on a massless rod of length ℓ, to which is attached by a pivot another identical rod with an identical mass m attached at the end, as shown in the figure. Using the angles θ_1 and θ_2 as generalized coordinates,

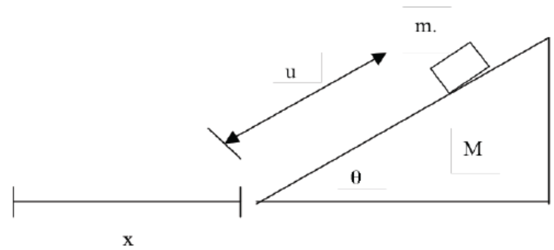


a) Find a Lagrangian for the system

b) Find an approximate Lagrangian that is appropriate for small oscillations and obtain from it the equations of motions when $\theta_1, \theta_2 << 1$.

c) Assuming that each angle varies as $\theta_{1,2} = A_{1,2}e^{i\omega t}$, find the frequencies ω for small oscillations.

18. A particle of mass m, at rest initially, slides without friction on a wedge of angle Θ and mass M that can move without friction on a smooth horizontal surface.



a) Find the Lagrangian using the coordinates (x, u) as shown.

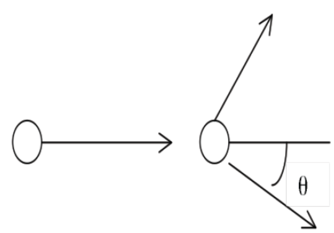
b) Derive the equation of motion from the Lagrangian.

c) What are the constants of motion for the system?

d) Describe the motion of the system.

e) What is the Hamiltonian H (P_x, p_u, x, u)?

19. A target particle of mass m is at rest in the reference frame of the laboratory. It is struck by a projective twice as massive as itself. The scattering is elastic.

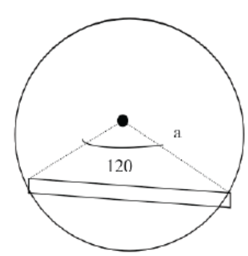


a) What is the largest "scattering angle" θ that the target particle can have after the collision?

b) At what scattering angle does the target particle have the most energy?

c) What is the maximum percentage of its energy that the incident particle can transfer to the scatterer (in lab system)?

20. A uniform rod slides with its ends inside a smooth (frictionless) vertical circle of radius a. The rod of uniform density and mass m subtends an angle of 120° at the center of the circle.



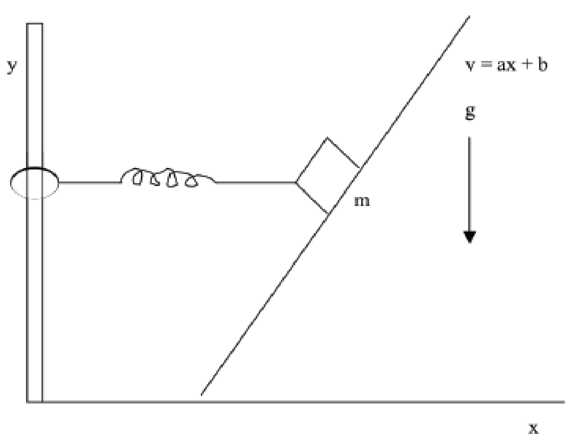
a) Compute the center of mass moment of inertia of the rod I_{cm} in terms of m and a (not the length of the rod).

b) Obtain the potential energy, the kinetic energy and the Lagrangian $L\left[\Theta(t),\dot{\Theta}(t);m,a,g\right]$ for this system, where Θ (t), the dynamical variable, is the instantaneous angular position of the rod relative to its equilibrium position and m, a, and g are constant parameters of the system.

c) Compare the dynamics of this system with that of a simple pendulum with mass M and length L (exact without any approximation such as the small oscillation approximation). Compare the parameters L and M with a and m.

d) Find the frequency of small oscillations of the system.

21. A mass, m, is attached to a spring of spring constant k that can slide vertically on a pole without friction, and moves along a frictionless inclined plane as shown in the figure. After an initial displacement along the plane the mass is released. Find an expression for the x and y position of the mass as a function of time. The initial displacement of the mass is x_0. You may assume that object never slides down the ramp so far that it strikes the floor.



22. An astronaut is on the surfaces of an spherical asteroid of radius r and mean density similar to that of the earth. On the earth this astronaut can jump about a height h of 0.5 m. If he jumps on this asteroid, he can permanent leave the surface.

a) Taking the radius of the earth as 6.4×10^6 m, find the largest radius the asteroid can have.

b) How fast could this asteroid rotated and not have the astronaut be flung away from the surface?

23. The relation between the differential scattering cross section in the laboratory and in the center of mass for two non-relativistic particles, m_1 and m_2, with m_2 initially at rest, being scattered by a central force is given by:

$$d\sigma / d\Omega)_L = d\sigma / d\Omega)_C (1+\gamma^2+2\gamma\cos\theta)^{3/2}/(1+\cos\theta),$$

where θ is the scattering angle in the center of mass system. The quantity γ is the ratio of the velocity of the center of mass to the velocity of particle one before the collision measured in the center of mass system.

a) Assume that the collision is inelastic with an energy loss of Q. Under this condition derive an expression for γ in terms of the mass of the two particles, Q and the total energy measured in the laboratory system, E_L.

b) Now assume that the two particles are charged, have equal mass, and the collision is perfectly elastic (Rutherford scattering). Derive the expression for the laboratory differential cross section in terms of the center of mass cross section and the laboratory scattering angle.

24. In 1956 G. Kwzmin introduced the simple axisymmetic potential

$$\Phi_k(R,z) = -\frac{GM}{\sqrt{R^2+(a+|z|)^2}},$$

as an approximate description of the potential of a disk-like galaxy.

a) Show that this potential can be thought of as the result of two point masses M placed at $R=0$, $z=\pm a$, generating the potential in the half-space $z < 0$ and $z > 0$ respectively.

b) Show that all the mass must be located on the $z = 0$ place and distributed according to the surface density

$$\sum\nolimits_k (R) = \frac{aM}{2\pi(R^2+a^2)^{3/2}} \quad .$$

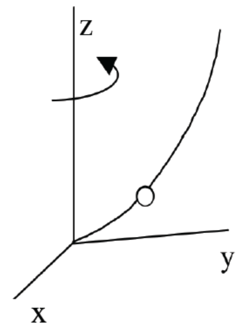
c) What is the total mass generating Φ_k?

25. A particle of mass m moves in a force field given by a potential energy V = Kr^8 where both K and s may be positive or negative.

a) For what values of K and s do stable circular orbits exist?

b) What is the relation between the period P of the orbit and the radius R of the orbit?

26. All parts of this question call for quick and brief answers.



a) A bead of mass m slides along a wire bent into a parabolic shape. The wire is pivoted at the origin and is spinning around the vertical. Identify suitable generalized coordinates and constraints to describe the bead's motion.

b) A system with generalized coordinates q_1, q_2 and q_3 is described by the Lagrangian $L(\dot{q}_1, \dot{q}_2, \dot{q}_3, q_3)$; that is, the Lagrangian is independent of q_1, q_2 and time (explicitly). What are the conserved quantities of this motion?

c) If the kinetic energy is a quadratic function of generalized velocities, $T = \sum_{k\ell} a_{k\ell} \dot{q}_k \dot{q}_\ell$, express $\sum_k \dot{q}_k (\partial T / \partial \dot{q}_k)$ in terms of T.

d) A hollow and a solid sphere, made of different materials so as to have the same masses M and radii R, roll down an inclined plane, starting from rest at the top. Which one will reach the bottom first?

e) Write down the Hamiltonian for a free particle in three-dimensional spherical polar coordinates.

27. All parts of this question call for quick and brief answers.

a) In a bound Kepler orbit, what is the ratio of the average kinetic energy to the average potential energy?

b) Write down the Lagrangian and the Hamiltonian for a free particle of mass m in three-dimensional spherical polar coordinates.

c) A system with generalized coordinates is described by a Lagrangian $L(\dot{q}_1, \dot{q}_2, \dot{q}_3, q_1)$; that is, the Lagrangian is independent of q_2 *and* q_3. What are the conserved quantities of this motion?

d) A hollow and a solid sphere, made of different materials so as to have the same masses and radii, roll down an inclined plane. Which one will reach the bottom first?

e) A satellite initially at a distance R_E *(= radius of the Earth)* above the Earth's surface is moved out to a distance $2R_E$. How does its orbital time period change?

28. A particle of mass m moves under the influence of an attractive force $F = -kr$, where r is the distance from the force center.

a) What are the conserved quantities of this three-dimensional motion?

b) Write down an expression for the total energy.

c) Either from (b) or otherwise, derive an orbit equation relating r to θ.

29. All parts of this question call for quick and brief answers.

a) Positronium is a bound state of an electron and positron. What is the effective mass of positronium?

b) At what point in a bound Kepler orbit is the speed of the satellite at its minimum?

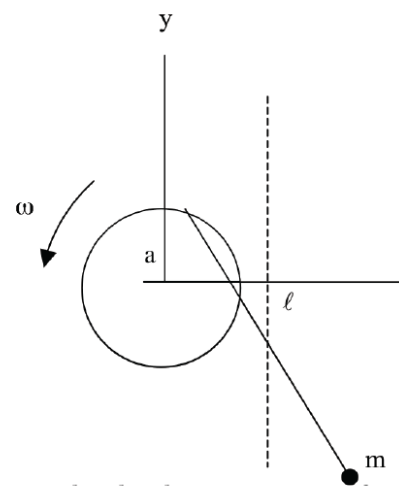
c) If you have two spheres of the same mass and radius, describe a non-destructive test by which you could distinguish between the one that is solid and the one that is hollow.

d) Which way is a particle moving from east to west in the southern hemisphere deflected by the Coriolis force arising from the Earth's rotation?

e) In a bound Kepler orbit, what is the ration of the average kinetic energy to the average potential energy?

f) Write down the Hamiltonian for a free particle of mass m in three-dimensional cylindrical coordinates.

30. A pendulum, free to swing in a vertical plane, has its point of suspension on the rim of a hoop rotating with constant angular speed ω. Write the Lagrangian and the resulting equations of motion in some suitable coordinates for this system.



31. Consider a system of N non-interacting particles in which the energy of each particle can assume two and only two distinct values: 0 and Δ. The total of the system is E.

a) Find the entropy S of the system as a function of E and N. Determine S in the thermodynamic limit, $N \gg 1$, $E \gg \Delta$.

b) Find the temperature as a function of E.

c) For what range of values of E is the temperature negative? Give a physical interpretation of your results.
Use formula: $\ln N! = N \ln N - N$

32. A crude estimate of the surface temperature of the earth is to assume that the clouds reflect a fraction of all sunlight, the rest being absorbed by the earth and reradiated. Treating the sun as blackbody at a temp T = 5800K, find the surface temp of the earth. You may assume the earth is an ideal absorber and that the rotation of the earth allows it to emit in all directions. The radius of the sun is 6.96 x 10 km and that of the earth is 6,400 km. The mean distance between the sun and the earth is 1.5 x 10 m.

33, A simplified model of diffusion consists of a one-dimensional lattice, with lattice spacing a, in which an "impurity" makes a random walk from one lattice site to an adjacent one, making jumps at time intervals Δt. After N jumps, the atom has taken N_1 steps to the right and N_2 steps to the left with $N = N_1 + N_2$. It is now located at $x = a(N_1 - N_2)$.

a) Find the probability the atom is at x, after N steps, given $N_1, N_2 >> 1$.

b) If a, Δt are taken infinitesimal the probability in (a) satisfies the diffusion equation

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$$

Find an expression for D in the terms of a, Δt.

34. Show that : $C_p = C_V + TV\alpha^2 / k_T$, where C_p and C_v are the heat capacities at constant pressure and volume, respectively, $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ is the coefficient of thermal expansion, and $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$ is the isothermal compressibility.

35. Lead has a molar mass of 207.2 g/mole. At 25°C and 1 atmosphere of pressure, it has an isothermal bulk modulus of B = 1.6 x 10^{10} Pa, a mass density ρ = 11.4 g/cm^3, and a coefficient of thermal expansion of 87 x 10^{-6}/°C. Its specific heat at constant pressure C_p = 128 J/kg-°C.

a) How big is the difference between C_p and its constant volume specific heat C_V?

b) The Law of Dulong and Petit states that the heat capacity of any solid at room temperature arises from the vibrations of the atoms (3N degrees of freedom), which can be calculated by treating the vibrations as a set of 3N classical harmonic oscillators. Does the Law of Dulong and Petit describe C_p or C_V? What would you predict the heat capacity of lead to be if this law is correct?

c) Find the Debye temperature of lead. How does the specific heat of lead vary with temperature for temperatures well below the Debye temperature?

36. Consider a monoatomic ideal gas of mass density r at temperature T, whose atoms have mass m. The number of atoms with velocities $\vec{v}$ in the velocity space volume element $d^3\vec{v}$ is given by the Maxwell-Boltzmann distribution

$$n(v)d^3v = \frac{\rho}{m}\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(\frac{-m(v_x^2+v_y^2+v_z^2)}{2mk_B T}\right)d^3v$$

a) What is the average velocity v_{avg}?

b) Derive the distribution of speed $P(v)dv$.

c) What is the most probable speed v^*?

d) Obtain expressions for the average $\bar{v}$, and root-mean-square speed v_{rms} and v^*, and rank them increasing order.
[Hint: $\int_0^\infty x^n e^{-x^2} dx = \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)$.]

37. A certain material is completely specified by its volume V and temperature T. It has an equation of state $p = AT^4$, where A is a constant independent of the volume. The heat capacity at fixed volume is measured to be BVT^3.

a) From dimensional analysis, B and A have the same units. Show that $B = 12A$.

b) Find the entropy of this material as a function of V and T.

c) If this material is cooled adiabatically and reversibly from 20K to 10K, by how much does the volume change?

38. Consider an ideal Fermi gas of spin 1/2 particles in 3-dimensional box. The number of particles per unit volume is n, the mass of the particles is m, and the energy of the particles is the usual $E = p^2/2m$. Assume that the temperature is quite low.

a) Find formulas for the Fermi energy E_F, Fermi wavevector k_F, and Fermi temperature T_F in term of m, n and constants such as h and k_B.

b) Find the total energy of the gas at zero temperature.

c) Show that the heat capacity at low temperature is proportional to T.

39. A collection of N spin 1/2 atoms are fixed in a solid. The atoms do not interact with each other. The magnetic moment of each atom is $\pm\mu_0$. If a magnetic field H is applied to the solid, each atom has an energy of $\pm\mu_0 H$.

a) Find the mean energy $\langle E \rangle$ in a magnetic field at a given temperature T.

b) Find the entropy of this collection.

c) The magnetization m of a solid is defined as the net magnetic moment per unit volume. The average magnetic moment is defined via $\langle E \rangle = - \langle M \rangle H$. For noninteracting moments, the magnetization typically obeys a Curie Law where $m = \chi_0 H/T$ for vanishingly small H. Find the value of the constant χ_0 for this problem.

40. An ideal non-relativistic gas of N spin -1/2 particles of mass m is confined to move in a two dimensional area of size $A = L^2$. Assume the energy levels are close enough to be described as a continuous density of states.

a) Find the density of states per unit area g(E).

b) Find the Fermi temperature in terms of the above quantities.

c) Find the total kinetic energy of the gas at zero temperature.

d) Find the chemical potential at all temperatures.

41. In the early universe, a chemical equilibrium between photons and e^+, e^- particles was achieved via the conversion process

$$e^+ e^- \leftrightarrow 2\gamma.$$

The energy of an electron or a positron is given by $E = mc^2 + (½)\, mv^2$. Using the fact that photon shave zero chemical potential, derive an equation describing the concentrations n^+ and n^- of positrons and electrons are related to each other at a given temperature.

42. A collection of N bosons is contained in a volume V. The spin of the particles is 0.

a) Find the temperature at which Bose condensation occurs.

b) Find how the number of particles in the lowest energy state varies with temperature below the condensation temperature.

43. The average energy of a system in thermal equilibrium is $\langle E \rangle$. Prove that the mean square deviation of the energy, $\langle (E - \langle E \rangle)^2 \rangle$ is given by

$$\langle (E - \langle E \rangle)^2 \rangle = k_B T C_V ,$$

where C_V is the heat capacity of the system at constant volume and k_B is the Boltzmann constant. Use this result to show that the energy of a macroscopic system may ordinarily be considered constant when the system is in thermal equilibrium.

44. Helium atoms can be absorbed on the surface of a metal. It requires an amount of work Δ to remove a helium atom from the metal surface to infinity. The motion of these helium atoms is restricted to two dimensions parallel to the metal surface. There is no mutual interaction between helium atoms. If such a system of helium atoms (consisting of only one layer) on a metal surface is in equilibrium with a gas of helium atoms (noninteracting) at a pressure P and temperature T, what is the mean number of atoms adsorbed per unit area of the metal surface?

45. Consider a container of volume 100 cm^3 containing a classical ideal gas at 1 atm pressure and 350K.

a) Find the number of particles.

b) Compute the mean kinetic energy of a particle in the gas.

c) Suppose one counted the number of particles in a small subvolume of size 0.1 micron on a side. What is the probability of finding no particles in this volume?

46. The pressure in a vacuum system is 10^{-3} mm Hg. The external pressure is 1 atm at 300 K. This is a pinhole in the vacuum system of area 10^{-10} cm^2. Assume that any molecule entering the pinhole goes through. Use an average molecular weight of air as 29 amu.

a) How many molecules enter the vacuum system each hour?

b) If the volume of the system is 2 liters, by how much does the pressure rise in 1 hour?

c) How long does it take for the pressure to rise to 750 mm Hg? Note: this is close to the pressure outside the vacuum tank.

47. Two phases of a pure material coexist along a line $p_{coex}(T)$. Use m_a as the molar mass of the substance. The latent heat of transformation (in J/mole) is L, and the molar volume of each phase are v_1 and v_2 respectively. Assume that phase 1 is the stable phase for temperatures lower than the temperature where they coexist.

a) What is known about the chemical potentials of each phase in the low temperature region where phase 1 exists, the high temperature region where phase 2 exists, and on the coexistence line?

b) Derive the Clausius-Clapeyron equation for the slope of the coexistence curve.

$$\frac{dp_{coex}}{dT} = \frac{L}{T(v_2 - v_1)}$$

c) If the higher temperature phase 2 can be treated as an ideal gas, and molar volume of phase 1 is so small compared to v_2 as to be neglected, find the dependence of the saturated vapor pressure on temperature.

48. The diatomic molecule HD has a set of rotational energy levels $E_\ell = k_B \theta_r \ell(\ell+1)$ which are $(2\ell+1)$ degenerate, and a vibrational spectrum $E_v = (k_B \theta_v)(n_v + \frac{1}{2})$. θ_r is called the rotational temperature, and θ_v the vibrational temperature.

a) Assuming the molecules do not interact among themselves, evaluate the partition sum for N atoms confined to a volume V. You may not be able to compute all the sums in closed form.

b) Evaluate the Helmholtz Free energy $F(T, V, N)$ in the limits $T \ll \theta_r, \theta_r \ll T \ll \theta_v$, and $T \gg \theta_v$. It will help to use the fact that when $k_B T$ is much smaller than the spacing of the energy levels, the sum can be computed as an integral.

c) Sketch, as accurately as you can, the behavior of $C_v(T)$ from 25K to 5000K, using a logarithmic scale in temperature. Assume that $\theta_r = 100K$ and $\theta_v = 3000K$.

49. Consider an equilibrium gas consisting of two types of atoms of masses m_1 and m_2, both obeying Maxwellian velocity distribution corresponding to the same temperature T,

$$f_i(\vec{v}_i)d\vec{v}_i = \left(\frac{m_i}{2\pi kT}\right)^{3/2} \exp[-m_i v_i^2 / 2kT]d\vec{v}_i\,,$$

where m_i is the mass of particle type i, and the f_i are the corresponding velocity distributions. Using the well-known identity

$$m_1\vec{v}_1^2 + m_2\vec{v}_2^2 = (m_1 + m_2)\vec{V}^2 + \mu\vec{v}^2\,,$$

where $\vec{V}$ is the center-of-mass velocity, $\vec{v}$ is the relative velocity and $\mu = m_1 m_2 /(m_1 + m_2)$ is the reduced mass, show that the distribution of relative velocities between the two types of atom is also Maxwellian, with distribution

$$f_{1,2}(\vec{v})d\vec{v} = \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp[-\mu v^2 / 2kT]d\vec{v}\,.$$

50. Consider a collection of N particles of spin 1/2 in a volume V. Compute the magnetic spin susceptibility of these particles if they can be treated as a nondegenerate gas, but

a) the spin is still treated quantum mechanically.

b) their magnetic moments are treated as a classical magnetic moment of $\mu_o = \sqrt{3}\mu_B$ (recall that $s^2 = 3/4$ for a spin 1/2), but that the moment can point in any direction, so $E = -\mu_o B\cos\theta$, with the component of the magnetization along B being given by $\mu_o\cos\theta$.

51. An object of heat capacity C is used as the cold thermal reservoir by a Carnot engine. The hot reservoir has an infinite heat capacity. During the operation of the engine to produce work, the temperature of the cold reservoir will slowly rise. Assume the starting temperature of the cold reservoir is T_c and the constant temperature of the hot reservoir is T_h.

a) To what temperature will the cold reservoir rise before the engine ceases to produce work?

b) How much heat flows into the cold reservoir until the engine stops producing work?

c) What is the total amount of work the engine can produce before the process stops?

52. A certain gas has an equation of state

$$P = b + \frac{nRT}{V}.$$

a) Find the value of $C_p - C_V$, where C_p and C_V are the heat capabilities at fixed pressure and volume.

b) Show that the constant volume heat capacity is not dependent on the volume. Specify how the heat capacity can depend on n and T.

c) This gas undergoes a process at fixed temperature where the volume is changed from V_i to V_f. Find the change in entropy in terms of the variables n, R, T, V_f and V_i.

53. Consider a classical gas composed of N particles of mass m that possess a permanent electric dipole moment p. The gas is subject to a uniform electric field of size E in the x-direction and is enclosed in a container of size V. The potential energy of a dipole is $-\vec{E} \cdot \vec{p}$.

a) Find the partition function.

b) Find the net polarization of the sample as a function of field strength, pressure, and temperature. The polarization P is defined as the net dipole moment per unit volume.

54. The energy spectrum of neutrinos (spin ½ massless particles) is $E=pc$. For a collection on N neutrinos confined to volume V, calculate

a) The Fermi wave vector.

b) The Fermi energy.

c) The total energy at T = 0.

d) The compressibility of the neutrino gas.

55. Consider an ideal Fermi gas of spin -1/2 particles in a 3-dimensional box. The number of particles per unit volume is n, the mass of the particles is m, and the energy of the particles is the usual $E=p^2/2m$. Assume that the temperature is quite low.

a) Find formulas for the Fermi energy E_F, Fermi wavevector k_F, and Fermi temperature T_F in terms of m, n and constants such as h and k_B.

b) Find the total energy of the gas at zero temperature.

c) If the magnetic moment of the particles is u_e, show that the paragmagnetic susceptibility x for low fields in the limit of zero temperature is given by

$$X = \frac{3n\mu_e^2}{2E_F}$$

56. Consider a system consisting of two particles, each of which can be in any one of three quantum states of respective energies 0, ε, and 3ε. The system is in contact with a heat reservoir at temperature T.

a) Write an expression for the partition function Z if the particles obey classical Maxwell-Boltzmann statistics and are considered distinguishable.

b) Write a similar expression if the particles obey Bose-Einstein statistics.

c) Write a similar expression if the particles obey Fermi-Dirac statistics.

57. A box of volume V hold blackbody radiation at a temperature T.

a) Show that the amount of energy stored in a small range of frequencies $(f, f + df)$ is given by

$$E_f \, df = \frac{3af^3}{e^{bf} - 1} df \ ,$$

and find all expressions for a and b in terms of V, T, K_B, h, c.

b) The Wien displacement law describes the relationship between the peak intensity in the blackbody energy distribution (1) and the temperature. Derive this relationship. Note: the function $x^3/(e^x-1)$ has a maximum at x = 2.82 approximately.

c) Find the total energy of the blackbody radiation and how it depends on V, T, h, c, k_B.

58. Consider a degenerate electron gas in which all electrons are considered to be highly relativistic, so that their energy $\epsilon = cp$ with p the magnitude of the momentum vector.

a) Calculate the density of states for this system.

b) Derive a relation, at zero temperature, between the particle density N/V and the Fermi energy ϵ_F for this system.

c) The internal energy of a nonrelativistic zero temperature 3D electron gas satisfies $U = \frac{3}{5}N\epsilon_F$. Derive the corresponding relation for the present relativistic electron gas.

59. According to quantum mechanics the possible energy levels of a simple harmonic oscillator are given by

$$E_n = (n+\frac{1}{2})\hbar\omega \text{ ,}$$

where n= 0, 1, 2, 3, $\cdots$, $\hbar$ is the Plank's constant (divided by, and ω is the angular frequency of the oscillation.

a) Calculate the average energy of the oscillator at a fixed temperature T.

b) The Einstein solid consists of $3N$ such harmonic oscillators, all at the same frequency ω. Find the heat capacity at constant volume of the Einstein solid and describe its behavior at high temperature and low temperature limits, respectively.

60. Derive the density of states for a uniform gas of bosons confined to an area A in two spatial dimensions. Show that Bose-Einstein condensation is not possible for this system. Finally, determine the system chemical potential, as a function of temperature and sketch it.

61. What is the height to diameter ratio of a right cylinder such that the inertia ellipsoid at the center of the cylinder is a sphere?

62. Obtain the equation of motion for a particle falling vertically under the influence of gravity when frictional forces obtainable from a dissipation function 1/2 kv^2 are present. Integrate the equation to obtain the velocity as a function of time and show that the maximum possible velocity for a fall from rest is $v=mg/k$.